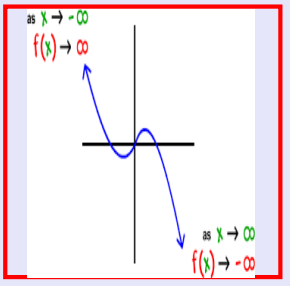
Math 245

Spring 2022

Lecture 25



Solve by square-root method:

$(5x+4)^2 + 36 = 0$

$(5x+4)^2 = -36$

$5x+4 = \pm\sqrt{-36}$

$5x+4 = \pm 6i$

$5x = -4 \pm 6i$

$x = -\frac{4}{5} \pm \frac{6}{5}i$ $\left\{-\frac{4}{5} \pm \frac{6}{5}i\right\}$

Make a perfect-square:

$x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = \left(x - \frac{7}{4}\right)^2 \Rightarrow x^2 - \frac{7}{2}x + \frac{49}{16} = \left(x - \frac{7}{4}\right)^2$

$\frac{1}{2} \cdot \frac{7}{2} = \frac{7}{4}$

Solve by Completing the square method:

$x^2 + 12x + 40 = 0$

$x^2 + 12x + 6^2 = -40 + 6^2$

$-40 + 36 = -4$

$\frac{1}{2} \cdot 12 = 6$

$(x + 6)^2 = -4$

Use S.R.M. $x + 6 = \pm\sqrt{-4}$

$x + 6 = \pm 2i$

$\boxed{x = -6 \pm 2i}$

$\{-6 \pm 2i\}$

Solve by the Completing the Square method:

$2x^2 - 3x - 5 = 0$

Divide by 2 to make the Leading Coef. 1.

$\frac{2}{2}x^2 - \frac{3}{2}x - \frac{5}{2} = 0$

$x^2 - \frac{3}{2}x - \frac{5}{2} = 0$

$x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{5}{2} + \left(\frac{3}{4}\right)^2$

$\frac{5 \cdot 8}{2 \cdot 8} + \frac{9}{16} =$

$\frac{40}{16} + \frac{9}{16} = \frac{49}{16}$

$\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

$\left(x - \frac{3}{4}\right)^2 = \frac{49}{16}$

Now S.R.M.

$x - \frac{3}{4} = \pm\sqrt{\frac{49}{16}}$

$x = \frac{3}{4} \pm \frac{7}{4}$

$x = \frac{3}{4} + \frac{7}{4} = \frac{10}{4} = \boxed{\frac{5}{2}}$

$x = \frac{3}{4} - \frac{7}{4} = \frac{-4}{4} = \boxed{-1}$

$\left\{-1, \frac{5}{2}\right\}$

Now Quadratic Formula

If $ax^2+bx+c=0$, $a\neq 0$, then

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ ← Discriminant

Ex: Solve $x^2-10x+29=0$

$a=1$ $b=-10$ $c=29$

$b^2-4ac=(-10)^2-4(1)(29)=100-116=-16$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-10)\pm\sqrt{-16}}{2(1)}=\frac{10\pm 4i}{2}=\frac{10}{2}\pm\frac{4}{2}i$$

$$=\boxed{5\pm 2i}$$

$\{5\pm 2i\}$

Solve by using the quadratic formula:

$$(2x-1)(3x+1)=21$$

FOIL, Simplify, write in $ax^2+bx+c=0$ form.

$6x^2+2x-3x-1-21=0$

$6x^2-x-22=0$

Identify a, b, c, and Compute b^2-4ac.

$a=6$ $b=-1$ $c=-22$

$b^2-4ac=(-1)^2-4(6)(-22)=1+528=529$

Now use the Q-Formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-1)\pm\sqrt{529}}{2(6)}=\frac{1\pm 23}{12}$$

$x=\frac{1+23}{12}=\frac{24}{12}=\boxed{2}$ $x=\frac{1-23}{12}=\frac{-22}{12}=\boxed{-\frac{11}{6}}$

$\left\{-\frac{11}{6},2\right\}$

The product of two consecutive integers is 90.

Find all such integers.

↳ x & $x+1$

$x(x+1)=90$ $\quad x^2+x=90$ $\quad x^2+x-90=0$

$a=1$ $\quad b=1$ $\quad c=-90$

$b^2-4ac=1^2-4(1)(-90)$

$=1+360=361$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{361}}{2}=\frac{-1\pm19}{2}$

$x=\frac{-1+19}{2}=\frac{18}{2}=9$ $\quad x=\frac{-1-19}{2}=\frac{-20}{2}=-10$

$x+1=10$ $\quad x+1=-9$

9 & 10 $\quad$ -10, -9

Find two consecutive even integers such that their product is 80.

↳ x & $x+2$

$x(x+2)=80$

x	$x+2$
8	10
-10	-8

8 & 10 OR -10 & -8

$x^2+2x+1^2=80+1^2$

$\frac{1}{2}\cdot 2=1$

$(x+1)^2=81$

use S.R.M.

$x+1=\pm\sqrt{81}$

$x=-1\pm9$

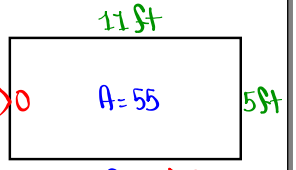
$x=-1+9=8$ $\quad x=-1-9=-10$

The length of a rectangular garden is 1ft longer than twice its width.

Area of the garden is 55 ft^2.

Find its dimensions.

5ft by 11ft



$A = 55$

$x(2x+1) = 55$

$2x^2 + x - 55 = 0$

$a=2 \quad b=1 \quad c=-55$

$b^2-4ac = 1^2-4(2)(-55) = 441$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{441}}{2(2)} = \frac{-1 \pm 21}{4}$$

$$x = \frac{-1+21}{4} = \frac{20}{4} = 5$$

$$x = \frac{-1-21}{4} = \frac{-22}{4} = \frac{-11}{2}$$

No school ⇒ Next Thursday

week after next week is Spring Break